Adaptive-Optimal Control of Constrained Nonlinear Uncertain Dynamical Systems using Concurrent Learning Model Predictive Control

Maximilian Mühlegg*, Girish Chowdhary†, Jonathan P. How ‡ and Florian Holzapfel§

A concurrent learning adaptive-optimal control architecture for aerospace systems with fast dynamics is presented. Exponential convergence properties of concurrent learning adaptive controllers are leveraged to guarantee a verifiable learning rate while guaranteeing stability in presence of significant modeling uncertainty. Nonparametric adaptive elements are incorporated to learn an appropriate basis for the uncertainty. The architecture switches to online-learned model based Model Predictive Control after an online automatic switch gauges the confidence in parameter estimates. Feedback linearization is used to reduce a nonlinear system to an idealized linear system for which an optimal feasible solution can be found online. It is shown that the states of the adaptively feedback linearized system stay bounded around those of the idealized linear system. Theoretical results and numerical simulations on a wing-rock problem with fast dynamics establish the effectiveness of the architecture.

I. Introduction

Model based optimal control of dynamical systems is a well studied topic. One of the most commonly used techniques for linear and nonlinear systems with constraints is model predictive control (e.g. Ref. 1–3). While this technique has been heavily studied and implemented for slower industrial processes, only in the past decade enough computational power has become available to enable online optimization for fast system dynamics typical for aerospace applications (some relevant demonstrations are in Ref. 4–9). MPC depends on a dynamic predictive model of the system. However, unaccounted modeling errors and dynamic variations in any real world scenario often result in an a-priori generated model of a system becoming obsolete or inaccurate. In such cases, the stability of an MPC approach cannot be guaranteed, especially if the underlying dynamics are nonlinear.[10] One way to deal with this is to estimate parameters of a model online, and then generate optimal controllers relying on the principle of certainty equivalence (see e.g. Ref. 11, 12) and online optimization techniques such as Model Predictive Control.[1,2] However, the main drawback of such indirectly-adaptive MPC methods is that it is difficult to guarantee stability, especially during parameter estimation transient phases. This is one major challenge in synthesizing algorithms for online adaptive-optimal control.[13]

Authors have studied adaptive-MPC architectures that rely on variants of the certainty equivalence principle. Fukushima et al. used the comparison principle to develop adaptive MPC for linear systems.[14] Adetola et al. considered adaptive MPC of linearly parameterized nonlinear systems and showed that one way to guarantee stability is to ensure that the initial parameter errors are within certain bounds.[15] Aswani et al. explored and experimented with the notion of safe-MPC by guaranteeing that control inputs are selected such that the system evolution is constrained to (approximations of) invariant reachable sets. They used an EKF for parameter estimation, which is known to not guarantee predictable and quantifiable learning rates under general operating conditions.[9,16] In general, while significant progress has been made, the results tend to be conservative, and the presence of learning transients prevent a general non-conservative solution to be formed.

On the other hand, adaptive control is one of the most well studied areas in control systems theory in which algorithms and techniques are developed for dealing with modeling uncertainties and disturbances. Direct adaptive control methods directly modify the system input to account for modeling uncertainties. In a certain light, these techniques

*Research Assistant, Institute of Flight System Dynamics, Technische Universität München, Munich, Germany.
†Postdoctoral associate, Massachusetts Institute of Technology, MA.
‡Richard C. Maclaurin Professor of Aeronautics and Astronautics, Aerospace Controls Laboratory (Director), Massachusetts Institute of Technology.
§Professor, Institute of Flight System Dynamics (Director), Technische Universität München, Munich, Germany.

could be viewed as model-free, in the sense that they do not focus on learning the system model, but rather on suppressing the uncertainty pointwise-in-time to minimize the instantaneous tracking error. Direct adaptive controllers can guarantee stability, even during harsh transients, however, they do not offer any long-term improvement due to model learning unless the system states are persistently exciting (PE; see e.g. Ref. 17). Furthermore, it is difficult to generate optimal solutions in presence of input and state constraints with direct adaptive control architectures.

Adaptive control literature also consists of hybrid-direct-indirect control architectures. For example, Duarte and Narendra, Lavretsky, and Chowdhary and Johnson have argued that modifying direct adaptive controllers to help focusing on learning the uncertainty improves performance (see e.g. Ref. 18–20). The power of these techniques is that they can handle harsh learning transients, while eventually guaranteeing learning of unknown model parameters subject to conditions on the system trajectories. It is natural therefore, to hypothesize that adaptive-optimal control algorithms can be devised that use provable hybrid adaptive control techniques to guarantee stability in the learning phase and then switch automatically to model-based optimal control algorithms (e.g. MPC) after sufficient confidence in estimated parameters has been gauged online. One such architecture is proposed in this paper and displayed in Figure 1. The main challenges in developing such an architecture include guaranteeing a verifiable learning rate for the uncertainty estimation such that the uncertainty is approximated in finite time before the architecture switches to the online learned model-based optimal controller guaranteeing stability before and after the switch, and guaranteeing that the architecture can switch back to the adaptive controller if ideal parameters of the system change.

In Ref. 21 we presented results of the CL-MPC architecture for nonlineaer uncertain dynamical systems assuming that the basis of the uncertainty was known. In Ref. 22 we presented a deadzone based switching algorithm as in Figure 2 for a CL-MPC switched system. In this paper, we extend both ideas to incorporate nonparametric adaptive elements as explored in Ref. 23 to learn an appropriate basis for the uncertainty and the associated parameters online.

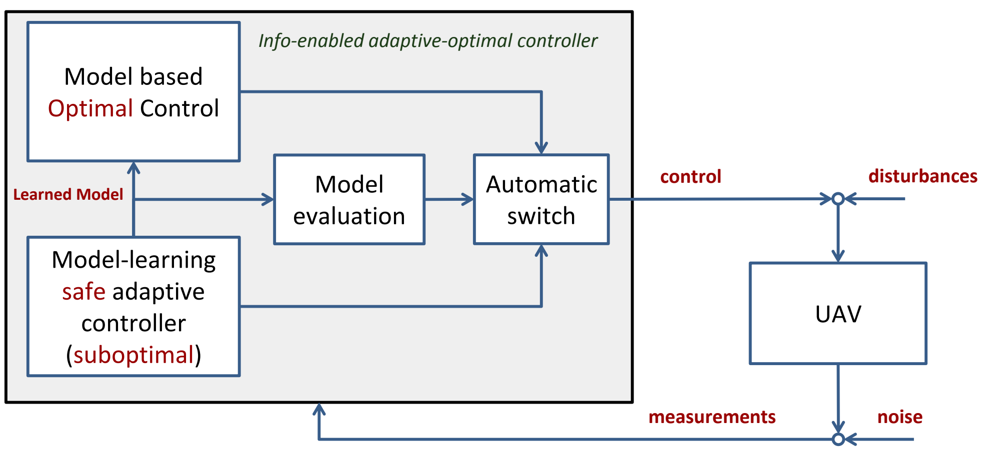

Figure 1. An adaptive-optimal control architecture. A learning-focused adaptive controller guarantees stability while learning uncertain system parameters. Once sufficient confidence has been gauged online in the estimated parameters, the architecture switches to using an online model-based controller, such as MPC. The resulting switched adaptive-optimal controller is guaranteed to be stable without being conservative about initial parameter errors.

References

[1] Bemporad, A. and Morari, M., "Robust model predictive control: A survey," *Robustness in identification and control*, edited by A. Garulli and A. Tesi, Vol. 245 of *Lecture Notes in Control and Information Sciences*, Springer Berlin / Heidelberg, 1999, pp. 207–226, 10.1007/BFb0109870.

[2] Nicolao, G. D., Magi, L., and Scattolini, R., *Nonlinear Model Predictive Control*, Vol. 26 of *Progress in Systems and Control Theory*, chap. Stability and Robustness of Nonlinear Receding Horizon Control, Birkhäuser, Basel-Boston-Berlin, 2000, pp. 3–22.

[3] Kerrigan, E. C., *Robust Constraint Satisfaction Invariant Sets and Predictive Control*, Ph.D. thesis, University of Cambridge, Department of Engineering, November 2000.

[4] Dunbar, W. B. and Murray, R., "Model Predictive Control of Coordinated Multi-vehicle Formations," *IEEE Conference on Decision and Control (CDC)*, 2002, pp. 4631–4636.

[5] Dunbar, W. and Murray, R., "Receding Horizon Control of Multi-Vehicle Formation: A Distributed Implementation," *IEEE Conference on Decision and Control (CDC)*, 2004.

[6] Dunbar, W., "Distributed Receding Horizon Control of Dynamically Coupled Nonlinear Systems," *IEEE Transactions on Robotics*, Vol. 52, 2007, pp. 1249–1263.

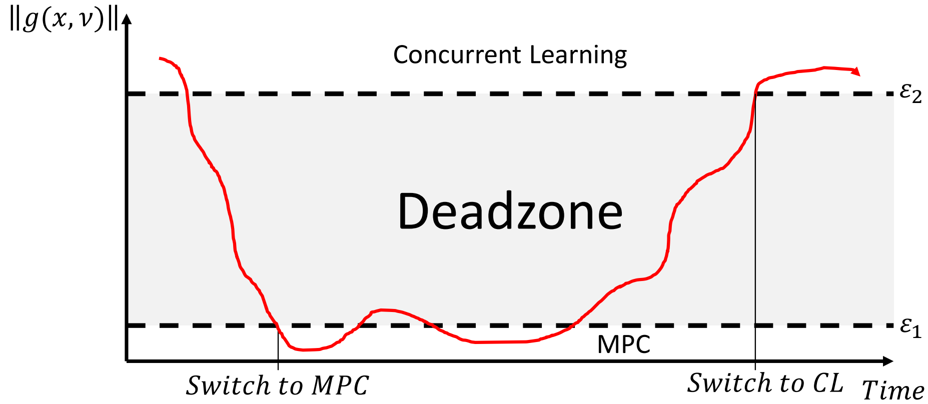

Figure 2. Graphical interpretation of a deadzone based switching algorithm between concurrent learning and MPC.

[7] Richards, A. and How, J. P., "Decentralized Model Predictive Control of Cooperating UAVs," *IEEE Conference on Decision and Control (CDC)*, Paradise Island, Bahamas, December 2004, pp. 4286–4291.

[8] Richards, A. G., *Robust Constrained Model Predictive Control*, Ph.D. thesis, Massachusetts Institute of Technology, Department of Aeronautics and Astronautics, Cambridge MA, February 2005.

[9] Bouffard, P., Aswani, A., and Tomlin, C., "Learning-based model predictive control on a quadrotor: Onboard implementation and experimental results," *Robotics and Automation (ICRA), 2012 IEEE International Conference on*, may 2012, pp. 279 –284.

[10] Rawlings, J., "Tutorial overview of model predictive control," *Control Systems Magazine, IEEE*, Vol. 20, No. 3, June 2000, pp. 38 –52.

[11] IOANNOU, P. and SUN, J., "Theory and design of robust direct and indirect adaptive-control schemes," *International Journal of Control*, Vol. 47, No. 3, 1988, pp. 775–813.

[12] Aström, K. J. and Wittenmark, B., *Adaptive Control*, Addison-Weseley, Readings, 2nd ed., 1995.

[13] Mayne, D. Q., *Nonlinear Model Predictive Control*, Vol. 26 of *Progress in Systems and Control Theory*, chap. Nonlinear Model Predictive Control: Challenges and Opportunities, Birkhäuser, Basel-Boston-Berlin, 2000, pp. 23–44.

[14] Fukushima, H., Kim, T.-H., and Sugie, T., "Adaptive model predictive control for a class of constrained linear systems based on the comparison model," *Automatica*, Vol. 43, No. 2, 2007, pp. 301 – 308.

[15] Adetola, V., DeHaan, D., and Guay, M., "Adaptive model predictive control for constrained nonlinear systems," *Systems and Control Letters*, Vol. 58, No. 5, 2009, pp. 320 – 326.

[16] Aswani, A., Humberto, G., S. S. S., and Tomlin, C., "Provably Safe and Robust Learning-Based Model Predictive Control," arxiv arXiv:1107.2487v2, UC Berkley, 2011.

[17] Boyd, S. and Sastry, S., "Necessary and Sufficient Conditions for Parameter Convergence in Adaptive Control," *Automatica*, Vol. 22, No. 6, 1986, pp. 629–639.

[18] Duarte, M. A. and Narendra, K. S., "Combined Direct and Indirect Approach to Adaptive Control," *IEEE Transactions on Automatic Control*, Vol. 34, No. 10, 1989, pp. 1071–1075.

[19] Lavretsky, E., "Combined/Composite Model Reference Adaptive Control," *Automatic Control, IEEE Transactions on*, Vol. 54, No. 11, nov. 2009, pp. 2692 –2697.

[20] Chowdhary, G. and Johnson, E. N., "Concurrent Learning for Convergence in Adaptive Control Without Persistency of Excitation," *49th IEEE Conference on Decision and Control*, 2010, pp. 3674–3679.

[21] Chowdhary, G., Mühlegg, M., How, J. P., and Holzapfel, F., "Concurrent Learning Adaptive Model Predictive Control," *2nd CEAS Specialist Conference on Guidance, Navigation and Control*, 2013, accepted.

[22] Chowdhary, G., Mühlegg, M., How, J. P., and Holzapfel, F., "A Concurrent Learning Adaptive-Optimal Control Architecture for Nonlinear Systems," *European Control Conference*, 2013, submitted.

[23] Kingravi, H. A., Chowdhary, G., Vela, P. A., and Johnson, E. N., "Reproducing Kernel Hilbert Space Approach for the Online Update of Radial Bases in Neuro-Adaptive Control," *Neural Networks and Learning Systems, IEEE Transactions on*, Vol. 23, No. 7, july 2012, pp. 1130 –1141.