Adaptive Flight Control with Guaranteed Convergence

Girish Chowdhary and Eric N. Johnson

Abstract We present a Concurrent-Learning approach for adaptive control that uses recorded data concurrently with instantaneous data for adaptive flight control. Concurrent learning can guarantee tracking error and weight error convergence to zero subject to a verifiable condition on linear independence of the recorded data; without requiring persistency of excitation in the system states. Simulation study of attitude tracking of an aircraft in presence of wing rock dynamics is used to highlight the benefits of the presented approach.

1 Introduction

Modern aerospace vehicles are expected to operate reliably in uncertain environments. Adaptive control is one approach that can be used to guarantee the stability of aircraft states in presence of significant modeling uncertainties. Consequently, adaptive flight control has been widely studied. For example, Calise [3], Johnson[16, 17, 15], Kannan [19, 18] and others have developed model reference adaptive controllers for both fixed wing and rotary wing Unmanned Aerial Systems (UAS). Cao, Yang, Hovaykiman, and other have developed the L1 adaptive control method [12, 6]. Lavertsky [22], Nguyen [28], and others have extended direct adaptive control methods to fault tolerant control and developed techniques in composite/hybrid adaptation.

Many of these approaches rely on a Model Reference Adaptive Control (MRAC) architecture, in which, the states of the aircraft are expected to follow the states of a chosen reference model which characterizes the desired transient response and stability properties. These adaptive control approaches use an adaptive element which

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attempts to capture the plant uncertainty through a weighted combination of a chosen basis. The weights of the adaptive element are updated online using a weight update law. The convergence of adaptive weights to ideal values which guarantee accurate parametrization of the plant uncertainty over the entire operating domain is a problem of considerable interest in adaptive control [2, 5]. However, most adaptive controllers update the weights in the direction of maximum reduction of a quadratic cost on the instantaneous tracking error by using using a gradient based adaptive law [23, 33]. For these methods, exponential tracking error and weight error convergence can only be guaranteed if the plant states are persistently exciting [1, 2, 27, 33].

In this paper we present an overview of a concurrent-learning adaptive control approach. Concurrent learning simulates memory in the adaptive law by using recorded data concurrently with instantaneous data and can guarantee exponential convergence in adaptive control without requiring persistency of excitation, subject to a verifiable condition on linear independence of the recorded data [8], [7]. In section 2, 3, and 4 the details of the concurrent learning adaptive control approach are presented. In section 5 we use a simulation study of attitude tracking in presence of wing rock dynamics to highlight the performance benefits of concurrent learning adaptive control. The paper is concluded in section 6.

2 Model Reference Adaptive Control

This section discusses the formulation of Model Reference Adaptive Control using approximate model inversion [15, 20, 25, 18]. Let $D_x \in \Re^n$ be compact, and Let $x(t) \in D_x$ be the known state vector, let $\delta \in \Re^k$ denote the control input, and consider the following system:

$$\dot{x} = f(x(t), \delta(t)), \tag{1}$$

where the function f is assumed to be continuously differentiable in $x \in D_x$, and control input δ is assumed to be bounded and piecewise continuous. The conditions for the existence and the uniqueness of the solution to 1 are assumed to be met.

Since the exact model 1 is usually not available or not invertible, we introduce an approximate inversion model $\hat{f}(x,\delta)$ which can be inverted to determine the control input δ:

$$\delta = \hat{f}^{-1}(x, \nu). \tag{2}$$

Where ν is the pseudo control input, which represents the desired model output $\dot{x}$ and is expected to be approximately achieved by δ. Hence, the pseudo control input is the output of the approximate inversion model:

$$\nu = \hat{f}(x, \delta). \tag{3}$$

This approximation results in a model error of the form:

$$\dot{x} = \nu(x, \delta) + \Delta(x, \delta) \tag{4}$$

where the model error $\Delta : \Re^{n+k} \to \Re^n$ is given by:

$$\Delta(x,\delta) = f(x,\delta) - \hat{f}(x,\delta). \tag{5}$$

A reference model can be designed that characterizes the desired response of the system:

$$\dot{x}_{rm} = f_{rm}(x_{rm}, r(t)), \tag{6}$$

where $f_{rm}(x_{rm}(t), r(t))$ denote the reference model dynamics which are assumed to be continuously differentiable in x for all $x \in D_x \subset \Re^n$. The command $r(t)$ is assumed to be bounded and piecewise continuous, furthermore, it is assumed that all requirements for guaranteeing the existence of a unique solution to 6 are satisfied. It is also assumed that the reference model states remain bounded for a bounded reference input.

A tracking control law consisting of a linear feedback part $u_{pd} = Kx$, a linear feedforward part $u_{crm} = \dot{x}_{rm}$, and an adaptive part $u_{ad}(x)$ is proposed to have the following form:

$$u = u_{crm} + u_{pd} - u_{ad}. \tag{7}$$

Define the tracking error e as $e(t) = x_{rm}(t) - x(t)$, then, letting $A = -K$ the tracking error dynamics are found to be [15, 7]:

$$\dot{e} = Ae + [u_{ad}(x,\delta) - \Delta(x,\delta)]. \tag{8}$$

The baseline full state feedback controller $u_{pd} = Kx$ is assumed to be designed such that A is a Hurwitz matrix. Hence for any positive definite matrix $Q \in \Re^{n\times n}$, a positive definite solution $P \in \Re^{n\times n}$ exists to the Lyapunov equation:

$$A^T P + PA + Q = 0. \tag{9}$$

Letting $\bar{x} = [x,\delta] \in \Re^{n+k}$The following two cases for characterizing the uncertainty $\Delta(x)$ are considered:

Case I: Structured Uncertainty: Let $\bar{x} = (x,\delta)$ and consider the case where it is known that the uncertainty is linearly parameterized with a known nonlinear basis $\Phi(\bar{x})$ is known. This case is captured through the following assumption:

Assumption 1. The uncertainty $\Delta(\bar{x})$ can be linearly parameterized, that is, there exist a matrix of constants $W^* \in \Re^{m\times n}$ and a vector of continuously differentiable functions $\Phi(\bar{x}) = [\phi_1(\bar{x}), \phi_2(\bar{x}), ..., \phi_m(\bar{x})]^T$ such that

$$\Delta(\bar{x}) = W^{*T}\Phi(\bar{x}). \tag{10}$$

In this case letting W denote the estimate W^* the adaptive law can be written as

$$u_{ad}(\bar{x}) = W^T \Phi(\bar{x}). \tag{11}$$

A large class of aircraft plants can be modeled in this manner.

Case II: Unstructured Uncertainty: Consider the case when it is only known that the uncertainty $\Delta(\bar{x})$ is continuous and defined over a compact domain $D \subset \Re^{n+k}$. In this case, a Radial Basis Function (RBF) Neural Network (NN) can be used as the adaptive element. In this case the adaptive element takes the following form

$$u_{ad}(\bar{x}) = W^T \sigma(\bar{x}). \tag{12}$$

where $W \in \Re^{n \times l}$ and $\sigma(\bar{x}) = [1, \sigma_2(\bar{x}), \sigma_3(\bar{x}), ..., \sigma_l(\bar{x})]^T$ is a vector of known radial basis functions. For $i = 2, 3..., l$ let c_i denote the RBF centroid and μ_i denote the RBF width then for each i The radial basis functions are given as

$$\sigma_i(x) = e^{-\|\bar{x}-c_i\|^2/\mu_i}. \tag{13}$$

Appealing to the universal approximation property of RBF NN (see reference [29] or [32]) we have that given a fixed number of radial basis functions l there exists ideal weights $W^* \in \Re^{n \times l}$ and a real number $\tilde{\varepsilon}(\bar{x})$ such that the following approximation holds for all $x \in D$ where D is compact:

$$\Delta(x) = W^{*T} \sigma(\bar{x}) + \tilde{\varepsilon}(\bar{x}), \tag{14}$$

and $\bar{\varepsilon} = \sup_{\bar{x} \in D} \|\tilde{\varepsilon}(\bar{x})\|$ can be made arbitrarily small when sufficient number of radial basis functions are used.

Figure 1 depicts the control architecture for MRAC control discussed in this section.

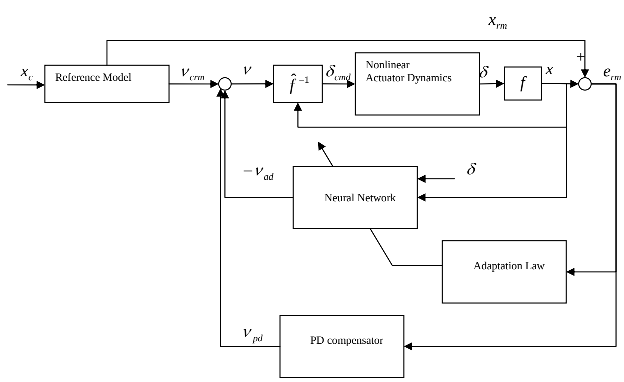

Fig. 1 Neural Network Adaptive Control using Approximate Model Inversion

2.1 Baseline Adaptive Law

For the case of structured uncertainty it is well known that in the presence of persistently exciting input the following adaptive law

$$\dot{\hat{W}} = -\Gamma_W \Phi(\bar{x}) e^T P \tag{15}$$

where Γ_W is a positive definite matrix of appropriate dimensions results in ultimate boundedness of the weights and $e(t) \to 0$ [26, 14, 33]. Equation 15 will be referred to as the baseline adaptive law. Furthermore, replacing $\Phi(\bar{x})$ with $\sigma(\bar{x})$ in equation 15 results in the baseline gradient based adaptive law for the case of unstructured uncertainty (case 2). For this case, the baseline adaptive law guarantees uniform ultimate boundedness of tracking error e [23, 21, 20].

The baseline adaptive law of equation 15 however, does not guarantee the boundedness of adaptive weights unless modification terms such as σ mod (see [14]), or e mod (see [26]) are added.Furthermore, in order to achieve transient performance guarantees further modifications must be made (see for example [4]). Furthermore, it is well known that global exponential tracking error ($e(t)$) and weight error ($\tilde{W}(t) = W(t) - W^*$) to zero is only guaranteed for the baseline adaptive law if and only if the plant states (and consequently the signal $\Phi(\bar{x}(t))$) are persistently exciting [27, 33].

Boyd and Sastry have shown that in the framework of MRAC, the persistency of excitation of $\Phi(x(t))$ can be guaranteed through the persistency of excitation of the tracking reference signal $r(t)$ [2, 33, 13]. Constant reference signals are not persistently exciting, nor are exponentially decaying reference signals. In fact a persistently exciting signal must contain as many spectral lines as the dimension of the basis of the uncertainty over a time interval (see reference [33] for a definition of persistently exciting signals). In flight control applications, enforcing such persistent excitation in the control system may not acceptable due to fuel and ride quality restrictions. Furthermore, since most flight controllers are event driven, it is hard to establish online whether the persistency of excitation condition is met.

On examining the baseline adaptive law of equation 15 we note that the law only uses instantaneous information for adaptation. In fact, it can be shown that this adaptive law attempts to minimize a quadratic cost on the instantaneous tracking error ($\frac{1}{2}e^T(t)e(t)$). The reliance on only instantaneous data is also reflected in the rank of $\dot{W}$, even though $\dot{W}$ is a matrix, its rank will be at-most one [15, 10]. This reliance on only instantaneous data is one reason why the baseline adaptive law must be persistently provided with information in order to guarantee exponential tracking error and weight error convergence. A concurrent learning adaptive law on the other hand, uses recorded and current data concurrently for adaptation. This reliance on memory of the concurrent learning law ensures that if the recorded data are sufficiently rich, then exponential tracking error and weight error convergence can be guaranteed without requiring persistency of excitation. In the following we present some key theorems that characterize these properties.

3 Guaranteed Exponential Convergence with Concurrent Learning Adaptive Control when the Structure of the Uncertainty is Known

In this section we present a concurrent learning adaptive controller which guarantees global exponential tracking error and parameter error convergence if a verifiable condition on the recorded data is met and the basis of the uncertainty is known.

Theorem 1. *Consider the system in equation 1, the control law of equation 7, the case of structured uncertainty (case 1). For the j^{th} recorded data point let $\varepsilon_j = \nu_{ad}(\bar{x}_j) - \Delta(\bar{x}_j)$, furthermore let p be the number of recorded data points $\Phi(\bar{x}_j)$ in the history stack matrix $Z = [\Phi(\bar{x}_1), ..., \Phi(\bar{x}_p)]$, such that $rank(Z) = m$, and consider the following weight update law:*

$$\dot{W}(t) = -\Gamma_W \Phi(\bar{x}(t)) e^T(t) P - \sum_{j=1}^{p} \Gamma_W \Phi(\bar{x}_j) \varepsilon_j^T(t). \tag{16}$$

Then the zero solution $(e(t), \tilde{W}(t)) \equiv 0$ of the closed loop system formed by the tracking error dynamics of equation 8 and the weight update law of equation 16 is globally exponentially stable. Furthermore, the rate of convergence is directly proportional to the smallest singular value of the history stack matrix Z.

Proof. A proof can be found in reference [7]. An equivalent theorem for a different class of plants is proved in [8].

Remark 1. In the above theorem it is shown that a verifiable condition on the linear independence of the recorded data is sufficient to guarantee that the zero solutions of the tracking error and the parameter error are globally exponentially stable. It is important to note that the imposed rank-condition on the recorded data ($rank(Z) = m$) is significantly different than a condition of persistency of excitation in the states. Firstly, this condition applies only to the recorded data, which is a small subset of all past states, whereas, the persistency of excitation condition applies to all past and future states. Secondly, since the rank of a matrix can be easily determined online, it is possible to verify whether this condition is met online, whereas it is impossible to determine whether a signal will be persistently exciting without knowing its future behavior. Hence, the rank-condition required to guarantee convergence when recorded data is concurrently used for adaptation with instantaneous data is less restrictive.

Remark 2. For evaluating the adaptive law of equation 16 the term $\varepsilon_j = \nu_{ad}(\bar{x}_j) - \Delta(\bar{x}_j)$ is required for the j^{th} data point. The model error $\Delta(\bar{x}_j)$ can be observed by using equation 5 noting that:

$$\Delta(\bar{x}_j) = \dot{x}_j - \nu(\bar{x}_j). \tag{17}$$

Since $\nu(\bar{x}_j)$ is known, the problem of estimating system uncertainty can be reduced to that of estimation of $\dot{x}$. In cases where an explicit measurement for $\dot{x}$ is not available, $\dot{x}_j$ can be estimated using an implementation of a fixed point smoother [11]. The details of this process are described in [7, 10].

4 Neuro-Adaptive Control with Guaranteed Boundedness with Concurrent Learning for Cases when the Structure of the Uncertainty is Unknown

When the structure of the uncertainty is unknown (case II in section 2), a RBF NN can be used as the adaptive element. The universal approximation theorem for RBF NN guarantees that for a given number of neurons an ideal set of weights W^* exists such that the difference $\tilde{\varepsilon}(\bar{x}) = \Delta(\bar{x}) - W^{*T}\sigma(\bar{x})$ is bounded by $\bar{\varepsilon}$ (see equation 14). Similar to the structured uncertainty case, the baseline adaptive law $\dot{\hat{W}} = -\Gamma_W \sigma(\bar{x})e^T P$ only guarantees that the weights approach the ideal weights if the plant states are persistently exciting. In this section we present a concurrent learning neuro-adaptive law that uses recorded data concurrently with instantaneous data to guarantee that the tracking error and weight errors remain bounded within a compact neighborhood of zero.

Theorem 2. *Consider the system in equation 1, the control law of equation 7, let $\bar{x}(0) \in D$ where D is compact, and the case of unstructured uncertainty (Case II). For the j^{th} recorded data point let $\varepsilon_j(t) = W^T(t)\sigma(\bar{x}_j) - \Delta(\bar{x}_j)$, furthermore let p be the number of recorded data points $\sigma(\bar{x}_j)$ in the matrix $Z = [\sigma(\bar{x}_1), ..., \sigma(\bar{x}_p)]$, such that $rank(Z) = l$. Then, the following weight update law*

$$\dot{W} = -\Gamma_W \sigma(\bar{x})e^T P - \Gamma_W \sum_{j=1}^{p} \sigma(\bar{x}_j)\varepsilon_j^T, \tag{18}$$

renders the tracking error e and the RBF NN weight errors $\tilde{W}$ uniformly ultimately bounded. Furthermore, the adaptive weights $W(t)$ will approach and remain bounded in a compact neighborhood of the ideal weights W^*.

Proof. A proof can be found in reference [7].

5 Trajectory Tracking in Presence of Wing Rock Dynamics

Modern highly swept-back or delta wing fighter aircraft are susceptible to lightly damped oscillations in roll angle known as "Wing Rock". Wing rock often occurs at low speeds and at high angle of attack, conditions commonly encountered when the aircraft is landing (see [30] for a detailed discussion of the wing rock phenomena). Hence, precision control of the aircraft in the presence of wing rock dynamics is critical to ensure safe landing. In this section we use concurrent learning control to track a sequence of roll commands in the presence of wing rock dynamics. Let ϕ denote the roll attitude of an aircraft, p denote the roll rate, δ_a denote the aileron control input, then a model for wing rock dynamics is [24]:

$$\dot{\phi} = p \tag{19}$$

$$\dot{p} = \delta_a + \Delta(x). \tag{20}$$

where $\Delta(x) = W_0 + W_1\phi + W_2 p + W_3|\phi|p + W_4|p|p + W_5\phi^3$. The parameters for wing rock motion are adapted from [31], [34], [35] they are $W_1 = 0.2314, W_2 = 0.6918, W_3 = -0.6245, W_4 = 0.0095, W_5 = 0.0214$. In addition to these parameters, a trim error is introduced by setting $W_0 = 0.8$. The linear part of the control law is given by $u_{pd} = -1.5\phi - 1.3p$, a second order reference model with natural frequency of $1 rad/sec$ and damping ratio of 0.5 is chosen, and the learning rate is set to $\Gamma_W = 2$. A simple inversion model has the form $\nu = \delta_a$. The adaptive controller uses the control law of equation 7. The simulation uses a time-step of 0.05 seconds. The concurrent learning controller uses the the weight update law of theorem 1, while the adaptive controller without concurrent learning uses the weight update law of equation 15. We present results for two different adaptive control scenarios.

5.1 Concurrent Learning Control of Wing Rock Dynamics with No Previously Recorded Data

In the first scenario, it is assumed that no previously recorded data points are available, and the concurrent learning adaptive controller actively populates the history stack through the simulation. An attitude tracking command of 1 *deg* and -1 *deg* is commanded between 15 to 17 *seconds* and 25 to 27 *seconds* respectively. The history stack is restricted to contain 30 recorded data points. New data points are selected and old data points replaced using an algorithm that ensures the minimum singular value of the history stack matrix $Z = [\Phi(\bar{x}_1), ..., \Phi(\bar{x}_p)]$ is maximized [9, 7]. The initial conditions are set to $\phi(0) = 1.2$ *deg* and $p(0) = 1$ *deg/sec*. Figure 2 compares the trajectory tracking performance of the adaptive controller of equation 7 with and without concurrent learning. The tracking performance is seen to improve significantly with concurrent learning, particularly when tracking the two steps in attitude. Figure 3 explicitly compares the tracking errors and the control commands for both cases. It can be seen that with concurrent learning, the tracking error rapidly approaches zero. Furthermore, the commanded control inputs remain comparable in magnitude. This is expected, as the improved performance of concurrent learning is through better estimation of the uncertainty. Figure 4 compares the evolution of the weights with and without concurrent learning. The weights rapidly converge to their true values with concurrent learning.

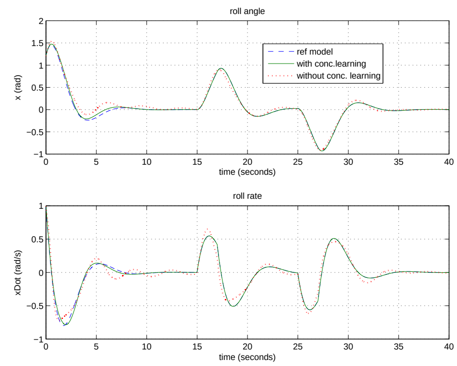

Fig. 2 Comparison of tracking performance of adaptive controller with and without concurrent learning. Note that concurrent learning adaptive controller exhibits significant tracking performance improvement, indicating the presence of long term learning in the adaptive controller.

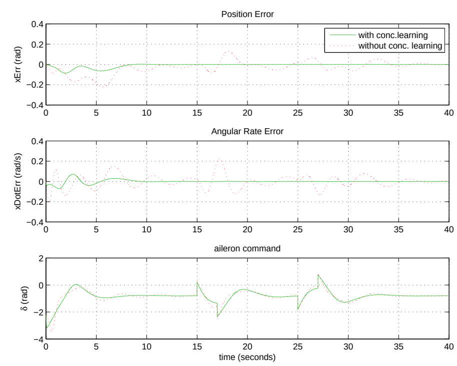

Fig. 3 Comparison of tracking errors of adaptive controller with and without concurrent learning.

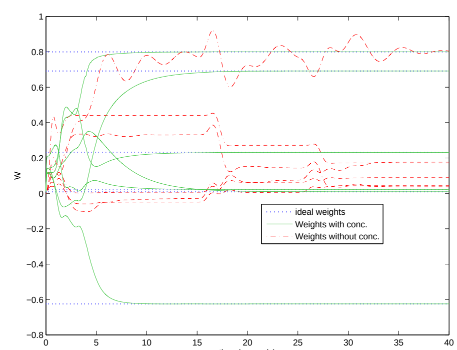

Fig. 4 Comparison of evolution of adaptive weights with and without concurrent learning. Note that the weights of the concurrent learning adaptive controller converge to the true weights, while the weights without concurrent learning do not converge.

5.2 *Concurrent Learning Control of Wing Rock Dynamics with Previously Recorded Data*

In the second scenario, it is assumed that a previously recorded history stack which satisfies the rank-condition (i.e. in this case $rank(Z) = 6$) is available. The history stack is populated with data points recorded from simulation in section 5.1. The initial conditions for this simulation are chosen to be $\phi = 3deg, p = 6deg/s$, and the controller is expected to track attitude commands that have double the magnitude. The initial conditions and tracking commands are more severe than those for the simulation in section 5.1. Figure 5 shows the tracking performance of the concurrent learning adaptive controller, while figure 6 shows the evolution of the weights. We note that the states track the commands with excellent accuracy and that the weights converge rapidly to their ideal values. This simulation shows that if a pre-recorded history stack satisfying the rank-condition is available for the system, then exponential convergence of tracking error and weight error to zero can be guaranteed for the adaptive controller even when the controller operates under different commands and initial conditions.

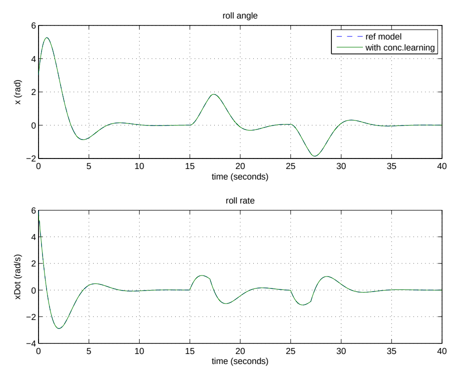

Fig. 5 Tracking performance of concurrent learning adaptive controller using a pre-recorded history stack. The controller tracks the commands with excellent accuracy, note that the history stack was recorded in a different simulation with different initial conditions, and different attitude commands.

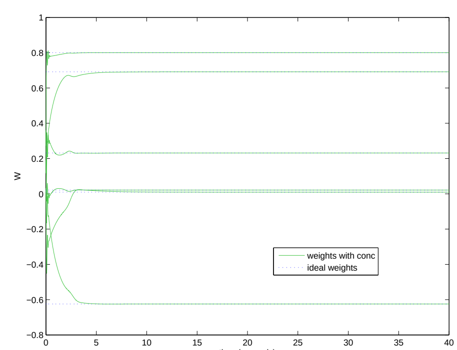

Fig. 6 Evolution of weights when using concurrent learning adaptive controller with a pre-recorded history stack. The weights converge rapidly to their true values.

6 Conclusion

In this paper a concurrent learning adaptive control approach that uses recorded and current data concurrently for adaptation was presented. The presented approach guarantees global exponential tracking error and weight error convergence to zero if the plant uncertainty is linearly parameterized and its structure is known. Furthermore, if the structure of the plant uncertainty is unknown, the concurrent learning adaptive control guarantees that the tracking error and weight errors are bounded within a neighborhood of zero. The concurrent learning adaptive control approach presented in this paper uses memory (in the form of recorded data) to alleviate the condition on persistency of excitation required by traditional adaptive laws that only use instantaneous data for adaptation. The effectiveness of the concurrent learning adaptive control approach was demonstrated on aircraft attitude tracking problem in the presence of wing rock dynamics.

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